ABSTRACT: This paper involves the results of the study undertaken on the partially prestressed concrete beams with bonded prestressing steel subjected to limited cycles of repeated loading. The flexural behavior of partially prestressed concrete beams has been investigated through an experimental program involving testing six full-scale simply supported beams with a clear span of (3000) mm. The goal of this study was to investigate the effect of limited repeated loading cycles on strength and serviceability (cracking and deformability) of partially prestressed concrete beams. Accordingly, these beams were divided into two identical sets. The first set consisted of three beams, one of tension-controlled, the second of transition-controlled, and the last one is compression-controlled. These beams are tested under static loading and considered as controlled beams. They were subjected to four-point bending by using two symmetrical concentrated static loads up to destruction. The loads were applied at the middle third of the clear span length. The second set consisted of exactly the same three beams as in set one but they were subjected to ten cycles of the repeated load. The range of repeated load was between “0.4 to 0.6” of ultimate load produced from the static test. After the ten cycles, the load was released and then the beams were subjected to monotonic static test until failure. Readings were made for strains in nonprestressed and prestressed steel, midspan deflections, crack widths and crack spacing at different loading increments. Findings show that partially prestressed compression-controlled beams were less sensitive to the repeated loading.

Keywords: Partial Prestressing, Bonded, Flexure, Deflection, Cracking, Repeated Load, Concrete Beams.

1. INTRODUCTION

Civil engineering structures may be exposed to different types of loads and among these loads is the repeated loading. The nature of loads that are exposed to bridges, offshore and multistory car parking is in fact repeated loading. Extensive theoretical and experimental studies of prestressed concrete beams over so many years have led to very well established methods for strength and serviceability design under static loads. However, the influence of repeated loading on deformability and cracking of prestressed concrete beams are still limited and rarely understood. Moreover, having a clear understanding and reasonable interpretation of the basics of deformability and cracking of concrete beams under different types of loading will improve the serviceability design and help to handle any difficult situation, particularly if not addressed by available codes and standards. For that research on the serviceability of concrete structures under repeated loading has become more essential in the last years. Concrete structures subjected to repeated loading experience higher deflection comparing to those exposed to static loading. These deflections were including significant permanent sets. With the increasing of the number of load cycles, the permanent deflections are also increased. This phenomenon has been observed by several researchers [1–5], however, suitable experimental information is still poor. The objective of the current research is to examine the influence of a limited number of repeated load cycles on the flexural characteristics of partially prestressed concrete beams with bonded strands.

2. EXPERIMENTAL PROGRAM

The experimental program involves testing six full-scale simply supported beams with overall dimensions of (200×300×3300) mm, which were divided into two groups. The first group consisted of three beams, PP-B-TC-S, PP-B-TRC-S, and PP-B-CC-S. According to the ACI code [6], these specimens were tension–controlled, transition–controlled and compression–controlled, respectively. Beams of the first group (I) were tested under monotonic static loading to collapse and regarded as controlled beams. The static loading was applied with increments ranged approximately between 2.5% to 5% of the predicted ultimate load. The deflection readings at the midspan section were monitored by mechanical
dial gauge at the end of each increment. The time consuming for testing one beam under static loading on average is between four to six hours depending on the load capacity of the tested beam.

The second group (II) is consisted of three beams, also, which identical to the beams of the first group but they were subjected to repeated loading as follows:

- Stage one: the minimum and maximum cyclic loads $P_{min}$ and $P_{max}$ were taken, respectively, as 40% and 60% of the ultimate load of the accompanying beams of group (Ithe ). The repeated static load was applied by increments each equals to 5% of the static ultimate load until reaching $P_{max}$ and then unloaded to $P_{min}$ by the same increments. Ten cycles of loading and unloading were implemented. The deflections, crack widths, crack spacing, crack numbers, and strand slip were recorded for each loading increment.

- Stage two: after ten cycles of loading, the load was released to zero. The residual deflections, crack width and strand slip were investigated.

- Stage three: in this stage, the beams were subjected to monotonic static loading until failure with increment equal to approximately 5% of the ultimate load of the accompanying beams of the group (I). During the third stage, the same measurements (deflections, crack width, number of cracks, the distance between cracks and strand slip) were taken for each load increments. The whole repeated load test took on average between 10 to 14 hours.

For the six rectangular pretensioned beams, the reference concrete mix was designed to achieve the target compressive strength of 40 MPa at 28 days for $(150×300)$ mm cylinder specimen. Concrete proportions by weight, cement: sand: gravel, were 1: 1.5: 2 with a maximum aggregate size of 9.5 mm and water/cement ratio by weight of 0.40. The beams were designed in such a way that the expected failure should occur due to flexure rather than shear; therefore, steel stirrups of Ø10 mm @ 100 mm c/c were used in shear spans. The steel stirrups were tied to two longitudinal bars of (10 mm) diameter at the top and to a different number of bars at the bottom depending on whether the beam is tension-controlled, transition or compression-controlled. In all prestressed concrete beams, two low-relaxation seven wires strands were used with target initial prestress $f_{pt}$ of 70% of the steel ultimate strength $f_{pu}$ - Yield stress of the longitudinal and the transverse mild steel bars was (570 MPa) and the characteristics strength of prestressed low-relaxation strand was (1862 MPa). Table 1 illustrates the PPR value and the nonprestressing tensile steel of each tested beams. Figure 1 shows the reinforcement details of each tested beam.

Two variables were investigated in this study, they are:

- Type of test (monotonic static and repeated loading)
- Partial Prestressing Ratio (PPR) which defined as the ratio of the ultimate resisting moment due to prestressing steel to the ultimate resisting moment due to the total tensile steel [7].

All beams were loaded in four-point bending using two symmetrical concentrated static loads applied at the middle-third of span length. All the measurements, such as midspan beam deflection, crack width and strand slip was recorded twice, immediately after the application of the load and 10 minutes later.

Deflection was measured at midspan of the beam using dial gauge of (0.01) mm accuracy. The cracks along the beams, the maximum crack width, average crack spacing and a number of cracks were measured during loading, sequentially. Strain in both types of steel (prestressed and nonprestressed) was recorded starting from stress transfer passing through load test until failure by using two electrical strain gauges fixed on each reinforcing bar.

**Table 1 Details of experimental beams**

<table>
<thead>
<tr>
<th>First group</th>
<th>PP-B-TC-S</th>
<th>PP-B-TRC-S</th>
<th>PP-B-CC-S</th>
</tr>
</thead>
<tbody>
<tr>
<td>Second group</td>
<td>PP-B-TC-R</td>
<td>PP-B-TRC-R</td>
<td>PP-B-CC-R</td>
</tr>
<tr>
<td>PPR</td>
<td>0.771</td>
<td>0.529</td>
<td>0.358</td>
</tr>
<tr>
<td>Mild steel in tension zone</td>
<td>2-Ø10</td>
<td>2-Ø10 + 2-Ø12</td>
<td>6-Ø12</td>
</tr>
</tbody>
</table>

Fig.1 Reinforcement details of experimental beams.

**3. EXPERIMENTAL RESULTS**

All beams in this study showed no fracture of steel or bond slip between concrete and any type of reinforcement (prestressed and nonprestressed). The failure was due to yielding of steel followed
by crushing of concrete at compression zone for tension-controlled beams or by crushing of concrete for compression-controlled beams.

### 3.1 Load - Deflection Response

The load-deflection curves for the tested beams are shown in Fig. 2. Every two curves for identical beams were depicted together (one for the specimen under monotonic static loading and the other for the specimen which exposed to ten cycles of repeated loading) to simplify the comparison process. Also, the ten load cycles behavior was magnified for more clarity.

**Fig. 2** Load-deflection curves for the tested beams.

Inspecting these diagrams, the following observations may be recorded:

- In a monotonic static test, tension-controlled and transition-controlled concrete beams, three distinctive points could be observed. These points are characterized by cracking, yielding and ultimate loads. Compression-controlled concrete beams are characterized by two points only which are the cracking and ultimate loads.

Thus, the clear yielding point cannot be distinguished.

- Prior to cracking all beams deformed elastically. After cracking, beams with low (PPR) developed approximately a linear load-deflection response.

- All beams experienced decreasing of stiffness after cracking, depending on the level of (PPR). Increasing the area of tension reinforcement produced a stiffer beam which led to a low rate of deflection progress versus the applied load.

- For tension-controlled and transition-controlled beams, almost nonprestressed steel in tension zone yielded prior to prestressed steel due to the far geometrical location of this steel relative to the member neutral axis.

- Beams with a low amount of nonprestressed steel (high level of PPR) exhibited more ductile behavior by their flatter and longer load-deflection curve.

- After ten cycles of repeated loading, when the load is decreased gradually to zero, all the beams suffered from permanent deflection. For specimen PP-B-TC-R which has high PPR, the permanent deflection was minimum (0.85 mm) and this may be attributed to the fact that the high ratio of the prestressing moment to the total nominal moment has the considerable effect to decrease the permanent deflection to a minimum value. Otherwise, beam PP-B-CC-R which has high nonprestressing steel (low PPR), the permanent deflection after load release was the higher and it was 1.55 mm. It seems that the small ratio of the prestressing moment to the total nominal moment has no ability to restrain the deformability of that beam.

Table 2 illustrates the load carrying capacities for both beam’s groups.

It can be seen that the difference in load carrying capacity for the identical beams is very small in both tests. The maximum difference was about 5% only. It can be seen also that both values of the ultimate load in both tests are very close to the theoretical value [8].

**Table 2 Theoretical and experimental load carrying capacities**

<table>
<thead>
<tr>
<th>Beam's labeling</th>
<th>$P_{\text{theo},S}$ (kN)</th>
<th>$P_{\text{exp},S}$ (kN)</th>
<th>$P_{\text{exp},T}$ (kN)</th>
<th>$P_{\text{exp},R}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>PP-B-TC-S,R</td>
<td>172</td>
<td>182</td>
<td>177</td>
<td>1.03</td>
</tr>
<tr>
<td>PP-B-TRC-S,R</td>
<td>213</td>
<td>230</td>
<td>243</td>
<td>0.95</td>
</tr>
<tr>
<td>PP-B-CC-S,R</td>
<td>209</td>
<td>310</td>
<td>312</td>
<td>0.99</td>
</tr>
</tbody>
</table>

Note: $P_{\text{theo},S}$ = theoretical ultimate load due to monotonic static loading [8]; $P_{\text{exp},S}$ = ultimate load produced from monotonic static test; $P_{\text{exp},R}$ =
ultimate load produced from the repeated static test.

Figure 3 characterizes midspan deflection at a maximum service load stage \((0.6 P_u)\) versus number of load cycles for the three tested beams under repeated loading.

The diagram shows that the variation of deflection with respect to load cycles can be represented by the following regression logarithmic expression:

\[
\Delta = \Delta_{ss} + \alpha \ln(N) \tag{1}
\]

where

\(\Delta\) = midspan deflection due to repeated static loading;
\(\Delta_{ss}\) = midspan deflection due to monotonic static loading immediately before the application of the repeated loading;
\(\alpha\) = parameter depends on PPR value; and
\(N\) = number of cycles of repeated static loading.

The above-mentioned expression can be regarded as a general equation because the number of beams which were tested under repeated loading in this study is not sufficient to get a satisfied value of \(\alpha\). It is required to test further beams with different PPR value to find a reliable value of the parameter \(\alpha\).

3.2 Strains in Steel Reinforcement

The strain in both types of steel has been documented starting from the moment of initial prestressing passing through stress transfer, testing until failure. Two strain gauges were fixed at each strand as well as nonprestressing tensile steel. Table 3 shows prestressed and nonprestressed steel strain at different stages for the tested beam under repeated loading.

Analyzing Table 3, the following findings may be noticed:

- The difference in strain between the first and the tenth loading cycles was very small in prestressed steel. It was between 1% and 2.8%. While the change was significant in nonprestressed steel, it was between 20% and 29%.
- The residual strain after ten cycles of repeated loading and load release to zero was very little in both types of steel. Actually, this strain is due to plastic deformation of concrete because neither prestressed steel nor nonprestressed steel reaches yielding when they subjected to these cycles.
- At failure, all strands in the three tested beams under repeated loading reached yielding (the stress was beyond 90% of the ultimate strength of prestressed steel).

Table 3 Progress of steel micro-strains during loading

<table>
<thead>
<tr>
<th>Beam's labeling</th>
<th>PP-B-TC-R</th>
<th>PP-B-TRC-R</th>
<th>PP-B-CC-R</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\varepsilon_p)</td>
<td>+6032</td>
<td>+6091</td>
<td>+5741</td>
</tr>
<tr>
<td>(\varepsilon_{ps1})</td>
<td>+7014</td>
<td>+7272</td>
<td>+6677</td>
</tr>
<tr>
<td>(\varepsilon_{ps10})</td>
<td>+7211</td>
<td>+7350</td>
<td>+6794</td>
</tr>
<tr>
<td>(\varepsilon_{ps\text{(release)}})</td>
<td>+6189</td>
<td>+6209</td>
<td>+5897</td>
</tr>
<tr>
<td>(\varepsilon_{ps})</td>
<td>+11299</td>
<td>+11128</td>
<td>+8783</td>
</tr>
<tr>
<td>(f_{ps\text{, (MPa)}})</td>
<td>1786</td>
<td>1786</td>
<td>1683</td>
</tr>
<tr>
<td>(\varepsilon_{se})</td>
<td>-786</td>
<td>-393</td>
<td>-236</td>
</tr>
<tr>
<td>(\varepsilon_{s1})</td>
<td>+480</td>
<td>+624</td>
<td>+1412</td>
</tr>
<tr>
<td>(\varepsilon_{s10})</td>
<td>+848</td>
<td>+746</td>
<td>+1735</td>
</tr>
<tr>
<td>(\varepsilon_{s\text{(residual)}})</td>
<td>-327</td>
<td>-218</td>
<td>+227</td>
</tr>
<tr>
<td>(\varepsilon_s)</td>
<td>+3021</td>
<td>+3211</td>
<td>+2652</td>
</tr>
<tr>
<td>(f_s\text{, (MPa)})</td>
<td>570</td>
<td>570</td>
<td>530</td>
</tr>
</tbody>
</table>

where: \(\varepsilon_p\) = effective prestrain; \(\varepsilon_{ps1}, \varepsilon_{ps10}\) = total strain in prestressed steel at the first and the tenth loading cycles, respectively; \(\varepsilon_{ps\text{(release)}}\) = strain in prestressed steel at load release; \(\varepsilon_{ps}, f_{ps}\) = prestressed steel total strain and stress at failure, respectively; \(\varepsilon_{se}\) = strain in nonprestressed steel at the beginning of test; \(\varepsilon_{s1}, \varepsilon_{s10}\) = nonprestressed steel strain at the first and the tenth loading cycles, respectively; \(\varepsilon_{s\text{(residual)}}\) = residual nonprestressed steel strain at load release; and \(\varepsilon_s, f_s\) = nonprestressed steel total strain and stress at failure, respectively.

3.3 Ductility
Ductility is one of the most important characteristics of structural concrete beams. It ensures gradual rather than brittle failure, providing a warning to the occupants before the collapse. If adequate ductility is not provided at the critical regions (plastic hinge zones), the member will be unable to develop the required inelastic rotation. The ductility is defined as "the ability of reinforced concrete members to sustain extensive inelastic deformations without excessive strength deterioration" [9]. Ductility is usually expressed for structural members and systems in terms of a deformation ductility ratio, where the deformation is described in terms of displacement or curvature.

Deflection ductility gives an indication of drift (ratio of lateral displacement to height) and is used in structural analysis. Curvature ductility is used to define member or section behavior at plastic hinges. Generally, ductility is measured by a ratio termed as ductility factor (μ) which is defined by the curvature (Ø), or displacement (Δ). Curvature ductility factor is:

\[ \mu = \frac{\Delta_u}{\Delta_y} \]

where:
- \( \Delta_u \) = the central deflection at failure.
- \( \Delta_y \) = the central deflection at yielding of tension steel reinforcement.

In terms of deflection, the ductility factor is:

\[ \mu = \frac{\Delta_u}{\Delta_y} \]

Table 4 illustrates the displacement ductility factor for both beam’s groups.

<table>
<thead>
<tr>
<th>Beam's labeling</th>
<th>PPR</th>
<th>Ductility factor, μ</th>
<th>( \mu_{Repeated} )</th>
<th>( \mu_{Static} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>PP-B-TC-S</td>
<td>0.771</td>
<td>8.0</td>
<td>1.23</td>
<td></td>
</tr>
<tr>
<td>PP-B-TC-R</td>
<td>0.529</td>
<td>4.56</td>
<td>0.77</td>
<td></td>
</tr>
<tr>
<td>PP-B-CC-S</td>
<td>0.358</td>
<td>2.40</td>
<td>0.89</td>
<td></td>
</tr>
<tr>
<td>PP-B-CC-R</td>
<td>0.23</td>
<td>2.13</td>
<td>(23%)</td>
<td></td>
</tr>
</tbody>
</table>

From Table 4, it was found that the ductility ratio is inversely proportional to the amount of reinforcement (nonprestressed or prestressed) of the beam. The case is different concerning the relation between PPR and the ductility ratio. The results show that the ductility ratio is directly proportional to PPR. On the other hand, repeated loading enhanced the ductility ratio of (PP-B-TC-R) beam compared with the counterpart beam (PP-B-TC-S) by about (23%). The situation is different concerning the ductility of beams (PP-B-TRC-R and PP-B-CC-R). The ductility is decreased by about (23%) and (11%), respectively.

3.4 Crack Spacing and Crack Width

The evaluation of flexural crack width and distances between cracks and how to control their development becomes very essential. Studies in this area are very limited due to the various parameters affecting crack width. Primary cracks initiated when the applied load reaches the cracking load. As the applied loading increased, additional cracks will appear. The number of cracks will be stabilized if the stress in concrete does not exceed the tensile strength irrespective of loading increase. This condition produces the absolute minimum crack spacing which happens at high stress in steel. This is called "stabilized minimum crack spacing". The maximum crack spacing is twice the minimum and is called "stabilized maximum crack spacing". The stabilized mean crack is the mean value of the above two extremes [10].

A number of expressions were developed by researchers to expect stabilized mean crack spacing but the most reliable equation was anticipated by Nawy [10] which is as follows:

\[ a_{cs} = k A_t/\sum o \]

where
- \( a_{cs} \) = stabilized mean crack spacing;
- \( k = 1.2 \) for pretensioned beams and 1.54 for posttensioned beams;
- \( A_t \) = area of concrete surrounding the tensile reinforcement bars; and
- \( \sum o \) = sum of the reinforcement element’s circumferences.

Recent studies on crack width control are supported on experimental studies. Based on these studies, some concluding points can be considered, mainly, using deformed bars is minimizing the crack widths, the maximum crack width is proportional to reinforcement stress, the distribution of reinforcement over the concrete tension zone will minimize the flexural crack widths, and the crack width at the extreme tensile concrete fiber is proportional to the concrete cover.

Gergely and Lutz [11] suggested the following equation to determine crack width:
\[ w_{cr} = 1.10276 \times 10^{-5} R_t \Delta f_s 3 d_c A_b \]  
\( (5) \)

where

- \( R_t = \) ratio of distances from tension face and from steel centroid to nethe utral axis;
- \( \Delta f_s = \) net tensile stress in reinforcing steel, MPa;
- \( d_c = \) thickness of concrete cover measured from center of bar closest to the concrete face to the tension concrete face, mm; and
- \( A_b = \) concrete area surrounding one bar, equal to total effective tension area of concrete surrounding reinforcement and having sathe me centroid, divided by a number of bars, mm.

Most cracks in all tested beams were propagated at the location of the middle third perpendicular to the longitudinal axis of the beam and extended to the tensile reinforcement. Mean crack spacing, crack width and number of cracks were monitored and measured throughout the test.

Figure 4 illustrates a number of cracks versus load cycles at the maximum service load range of 0.6 \( P_u \) for the beams tested under repeated loading.

These diagrams indicate that the number of cracks is stabilized from the first load cycle for the compression-controlled beam (PP-B-CC-R) while for the other beams; it is stabilized at the fifth loading cycle.

\[ \text{Table 5 Mean crack spacing for tested beams} \]

<table>
<thead>
<tr>
<th>Beam's labeling</th>
<th>Monotonic static loading</th>
<th>Repeated static loading</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Theor. ( a_{cs}, ) (mm)</td>
<td>Exp. ( a_{cs}, ) (mm)</td>
</tr>
<tr>
<td>PP-B-TC-S,R</td>
<td>185</td>
<td>110</td>
</tr>
<tr>
<td>PP-B-TRC-S,R</td>
<td>115</td>
<td>100</td>
</tr>
<tr>
<td>PP-B-CC-S,R</td>
<td>85</td>
<td>100</td>
</tr>
<tr>
<td></td>
<td>at 1st cycle</td>
<td>at 10th cycle</td>
</tr>
<tr>
<td></td>
<td>140</td>
<td>115</td>
</tr>
<tr>
<td></td>
<td>125</td>
<td>120</td>
</tr>
<tr>
<td></td>
<td>115</td>
<td>115</td>
</tr>
</tbody>
</table>

Table 6 shows the crack width of the beams tested under static loading and the counterpart beams that were exposed to repeated loading. As in Table 4, the results are at service load stage 0.6 \( P_u \) and for the repeated test, the table shows crack width at the first and tenth loading cycles.

From Table 6, the following observations may be reported:

- Crack width calculated using Gergely and Lutz expression, for specimens under monotonic static loading, gave good agreements of results comparing to the experimental results.
- Only for beam PP-B-TC-R, the crack width is influenced by repeated loading. It is increased by about 50% during the ten loading cycles.

\[ \text{Table 6 Crack width at the loading stage of 0.6 } P_u \]

<table>
<thead>
<tr>
<th>Beam's labeling</th>
<th>Monotonic static loading</th>
<th>Repeated static loading</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Theor. ( w_{cr}, ) (mm)</td>
<td>Exp. ( w_{cr}, ) (mm)</td>
</tr>
<tr>
<td></td>
<td>at 1st cycle</td>
<td>at 10th cycle</td>
</tr>
<tr>
<td>PP-B-TC-S,R</td>
<td>0.05</td>
<td>0.03</td>
</tr>
<tr>
<td>PP-B-TRC-S,R</td>
<td>0.10</td>
<td>0.17</td>
</tr>
<tr>
<td>PP-B-CC-S,R</td>
<td>0.15</td>
<td>0.12</td>
</tr>
<tr>
<td></td>
<td>0.10</td>
<td>0.15</td>
</tr>
<tr>
<td></td>
<td>0.15</td>
<td>0.15</td>
</tr>
<tr>
<td></td>
<td>0.10</td>
<td>0.10</td>
</tr>
</tbody>
</table>

Figure 5 shows all the tested beams at failure. It can be observed that the limited repeated loading has no significant effect on the number of cracks, cracks propagation and crack alignment. The figure shows also that, in compression-controlled beams, the cracks were of flexural and flexural-shear types. On the other hand, in tension-controlled beams, the cracks were only of a flexural type and they were approximately located at the pure bending moment zone (the middle third).
Fig. 5 Crack propagation of tested beams.

4. CONCLUSIONS

The following conclusions can be drawn:
1. The ultimate load carrying capacities of partially prestressed concrete beams exposed to limited cycles of repeated loading was approximately the same as the capacities of accompanying beams under monotonic static loading.
2. The response of the load-deflection curve under repeated loading can be represented by the envelope curve of monotonic static loading.
3. The increasing rate of deflection, crack width and crack spacing was significant in the first five load cycles especially for the tension-controlled beam.
4. All beams in this study showed no bond slip between concrete and any type of reinforcement. The failure was due to steel yielding followed by crushing of concrete at compression zone for tension-controlled beams or by crushing of concrete for compression-controlled beams.
5. Partially prestressed concrete beams with a large area of nonprestressing tension reinforcement, (i.e., low level of PPR), were less sensitive to repeated loading.
6. Increasing the amount of nonprestressing steel enhanced the flexural characteristics and the ability of partially prestressed concrete beams to resist the effect of repeated loading.
7. The residual deflection depends on the value of PPR, as PPR increased the residual deflection decreased. Accordingly, in beams with high nonprestressing steel (i.e., low level of PPR) the prestressing force has no ability to restrain the deformability of the beam in a small range.
8. All strands in all the beams attained yielding to strength.

5. REFERENCES