EVALUATING THE STATIC AND DYNAMIC MODULUS OF ELASTICITY OF ROLLER COMPACTED RUBBERCRETE USING RESPONSE SURFACE METHODOLGY

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Received: 9 Nov. 2017, Revised: 7 Dec 2017, Accepted: 25 Dec. 2017

ABSTRACT: In this study, response surface methodology (RSM) was used to evaluate the effect of partial replacement of fine aggregate with crumb rubber (CR), and the addition of nano silica (NS) by weight of cement in roller compacted concrete (RCC) pavement. Roller compacted rubbercrete (RCR) is used as the terminology for RCC where fine aggregate is partially replaced with crumb rubber. The experiments were designed and analysis executed using the historical data model type. After executing the experimental works, regression analysis was used to develop models for predicting the static and dynamic modulus of elasticity (MOE) of RCR. The RSM regression analysis showed that both static and dynamic MOE decreases with increase in CR replacement level, and increases with increment in addition of NS up to 2%. Therefore CR increases the flexibility of RCR while NS increases its stiffness. The analysis of variance for the developed models showed that the static MOE of RCR can be predicted using cubic model type while the dynamic MOE can be predicted using quadratic model type, with all the models having high degree of correlation and was in agreement with the experimental data.

Keywords: Crumb rubber; Nano silica; Static modulus of elasticity; Dynamic modulus of elasticity; Roller compacted rubbercrete; Response surface methodology.

1. INTRODUCTION

Roller compacted concrete (RCC) causes a major development to the mass concrete construction industries by fastening and easing the traditional methods of placement, compaction, and consolidation [1]. In simple terms, RCC can be defined as a dry lean concrete of zero slump consistency that is constructed using a similar process as in pavement construction [2]. Therefore RCC must be dry enough to be able to support the weight of vibratory roller in its fresh state so as to achieve full compaction and consolidation and yet wet enough to allow for adequate mortar distribution during mixing and placement [3, 4].

Due to the way RCC pavement is placed, compacted and consolidated, steel reinforcement, dowel bars or tie rods cannot be placed [5, 6]. Therefore loads and stresses are transferred using aggregate interlock down to the bottom layers [7]. Stresses in RCC pavement can be due to loads and due to adiabatic temperature caused mainly by the heat of hydration of cementitious materials during mixing which can result to thermal cracking. There is a direct proportionality between the thermal stresses and modulus of elasticity (MOE) of RCC pavement, thus lower elastic modulus in RCC is of higher desirability. However, factors which affect the MOE of RCC pavement include type and volume of aggregate, water/cement ratios and rate of hydration [3]. MOE can also be used for estimation of the bending deflections and calculating the deformation of the RCC pavement [8].

On the other hand, response surface methodology (RSM) is commonly used statistical and mathematical technique used for analysing and developing models between one or more independent variables and responses [9, 10]. In addition, RSM can be used for model multi-objective optimization by setting defined desirable goals based on either responses or variables [9]. Mohammed, et al. [11], has utilized RSM to model the compressive strength of concrete containing paper mill as additives. Mtarfi, et al. [12], have optimized and developed model for predicting mortar compressive strength with RSM. Güneyisi, et al. [13] have developed models and optimized high-performance concrete by minimizing the durability factors and maximizing compressive strength utilizing metakaolin and fly ash as variables. Mohammed, et al. [14] developed mix design model for self-compacting engineered cementitious composites (SC-ECC) using RSM.
They also optimized ECC mixtures by maximizing modulus of elasticity and energy absorption. Mohammed, et al. [15] also optimized rubbercrete mixtures using RSM. Mohammed and Adamu [16], also optimized crumb rubber and nano silica contents in RCC pavement using RSM by maximizing strengths and minimizing water absorption.

In this study, RSM was used to evaluate the effect of CR and NS on the static and dynamic MOE of RCC pavement, and develop relationship between them. CR was used as partial replacement to fine aggregate to increase flexibility of RCC pavement, and Nano silica was used as addition by weight of cementitious materials to mitigate loss strength in RCC pavement with incorporation of CR. Roller compacted rubbercrete (RCR) terminology was used throughout this study to denote RCC where crumb rubber was used as a partial replacement to fine aggregate.

2. MATERIALS AND METHODS

2.1 Materials

Type 1 ordinary Portland cement conforms to the requirements of ASTM C150 was used. Natural sand with a specific gravity of 2.65, absorption of 1.24% and fineness modulus of 2.68 was used as fine aggregate. Two nominal maximum sizes of coarse aggregates have been used to achieve the desired combined aggregate gradation. They are 19 mm size with a specific gravity of 2.66 and water absorption of 0.48% and 6.35 mm size having a specific gravity of 2.55 and water absorption of 1.05%. Three different CR sizes were combined so as to obtain similar gradation to fine aggregate. Several trial sieve analysis has been conducted in accordance with the requirements of ASTM D 5644 and a combination of 40% mesh 30 (0.595 mm), 40% 1-3 mm and 20% 3-5 mm has been selected. In order to achieve the recommended combined aggregate gradation and a more cohesive paste, the percentage of materials finer than 75 µm should be between 2% and 8% of the total aggregates, and materials such as naturally occurring non-plastic silt, fine sand or Pozzolanas can be used [6, 17]. In this study, class F fly ash has been used as mineral filler. Strong hydrophobic nano silica of size 10-25 nm has been used as an additive to the cement.

The geotechnical approach to ACI 211.3R [18] was used for mix design. 13% cement content was selected based on target flexural strength of 4.8 MPa at 28 days.

2.2 Experimental Design and Test Methods

2.2.1 Response Surface Methodology

The Response surface methodology (RSM) was used to develop relationships between static/dynamic MOE of RCR with the variables (CR and NS). The Design of Experiment (DOE) version 11 software was used for RSM analysis. The historical data model type was used to design the experiments and executing the analysis. The variation CR was 0%, 10%, 20%, and 30% by volume of fine aggregate and Nano silica was 0%, 1%, 2% and 3% by weight of cementitious materials. The total runs (based on actual value) and the variable constituent materials for each run are shown in Table 1. Other constituent materials which were fixed for each mix include; cement (268.69 kg/m³), filler (103.76 kg/m³), 19 mm coarse aggregate (415.03 kg/m³), 6.35 mm coarse aggregate (416.85 kg/m³), and water (98.24 kg/m³).

2.2.2 Sample Preparations and Experimental Programs

For each mix, a total of six 150 mm by 300 mm cylinders were produced out of which three were tested for compressive strength at 28 days and three were tested for static modulus of elasticity (MOE) according to ASTM C469 at 28 days. The static MOE is computed using the relation shown in Eqn 1

\[ E_c = (\sigma_2 - \sigma_1)/(\varepsilon_2 - 0.00005) \]  

where \( E_c \) = modulus of elasticity; \( \sigma_2 \) = stress equivalent to 40% of ultimate compressive force; \( \sigma_1 \) = strain corresponding to a longitudinal strain of 50x10^-6; \( \varepsilon_2 \) = longitudinal stress corresponding to \( \sigma_2 \).

The dynamic MOE was calculated from 150 mm × 150 mm × 150 mm cubes after age 28 days curing using Eqn 2 as given by Lamond [19]. Firstly the ultrasonic pulse velocity of the mixes was determined in accordance with ASTM C597-09 using the PUNDIT with a transducer of 54 KHz, and the hardened density was determined in accordance with BS EN 12390-7:2009.

\[ E_0 = \frac{(UPV)^2\rho(1+\mu)(1-2\mu)}{(1-\mu)} \]  

where \( E_0 \) = dynamic modulus of elasticity; \( UPV \) = pulse velocity; \( \rho \) = density; \( \mu \) = Poisson's ratio.
where $E_D$ is the dynamic modulus of elasticity (DMOE) for RCR; $\rho$ is the hardened density in kg/m$^3$; $\mu$ is dynamic Poisson ratio. The value of $\mu$ was assumed to be 0.2 for calculation of DMOE [20, 21].

### Table 1. Mixtures constituent materials

<table>
<thead>
<tr>
<th>Run</th>
<th>A: Crumb Rubber (%)</th>
<th>B: Nano Silica (%)</th>
<th>Cement Nano Silica</th>
<th>Filler Fine aggregate</th>
<th>Crumb rubber Static MOE (GPa)</th>
<th>Dynamic MOE (GPa)</th>
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<tr>
<td>1</td>
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<td>1148.05</td>
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<td>0</td>
<td>103.76</td>
<td>918.44</td>
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<td>2.69</td>
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<td>2.69</td>
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<td>918.44</td>
</tr>
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</table>

### 3. RESULTS AND DISCUSSION

#### 3.1 Analysis of Variance

The results of the responses, which were developed based on the mix design developed by the RSM, were presented in Table 1. These results were used for developing the models and least squares regression analysis.

The result of ANOVA for the developed response models is presented in Table 2. The significance of each variable and the responses are evaluated using the 95% confidence interval which corresponds to probability P-value<0.05. All the models have P-values less than 0.05, meaning they are all significant models at 95% confidence level. The model type for the static MOE was cubic with all the terms significant except CR*NS, CR$^2$, CR$^2$*NS, and CR*NS$^2$. The quadratic model type was more suitable for the dynamic MOE. All the model terms significant except CR*NS. The lack of fit for both models was not significant; this implies that the experimental results fitted well into the models. The adequacy of the models can also be checked by its degree of correlations, as seen from Table 2, the static MOE and Dynamic MOE models has 98% and 96% correlation degrees respectively with only 2% and 4% respectively of their data not explained by the model which is in agreement with their low coefficient of variation (CoV) and standard deviation (SD), meaning the experimental data perfectly fit into the model. In addition, for both models, the predicted R$^2$ are in agreement with the adjusted R$^2$ as their differences are less than 0.2.

The developed response models for static and dynamic MOE with the insignificant terms removed using backward regression analysis and hierarchical terms added afterwards are presented in Eqn 3a and 3b respectively. From the developed equations, the negative and positive signs before the terms denote the antagonistic and synergistic effects of the individual variables on the elastic modulus of RCR.


Table 2. ANOVA results of developed models

<table>
<thead>
<tr>
<th>Variable</th>
<th>Factors</th>
<th>F-Values</th>
<th>P-Values</th>
<th>R²</th>
<th>Adjusted R²</th>
<th>Predicted R²</th>
<th>SD</th>
<th>C.V (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Modulus of Elasticity (GPa)</td>
<td>Model</td>
<td>37.39</td>
<td>&lt;0.0001</td>
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<tr>
<td></td>
<td>CR</td>
<td>166.27</td>
<td>&lt;0.0001</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>NS</td>
<td>8.10</td>
<td>0.0216</td>
<td></td>
<td></td>
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<tr>
<td></td>
<td>CR*NS</td>
<td>0.17</td>
<td>0.6891</td>
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<td></td>
<td>CR²</td>
<td>0.033</td>
<td>0.8597</td>
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<td>NS²</td>
<td>12.74</td>
<td>0.0073</td>
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<tr>
<td></td>
<td>CR*NS²</td>
<td>0.69</td>
<td>0.4305</td>
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<tr>
<td></td>
<td>CR³</td>
<td>74.33</td>
<td>&lt;0.0001</td>
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<td></td>
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<tr>
<td></td>
<td>NS³</td>
<td>11.84</td>
<td>0.0088</td>
<td></td>
<td></td>
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<tr>
<td></td>
<td>Lack of Fit</td>
<td>16.79</td>
<td>0.0573</td>
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<tr>
<td>Dynamic Modulus of Elasticity (GPa)</td>
<td>Model</td>
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<td>&lt;0.0001</td>
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<td></td>
<td>CR</td>
<td>236.43</td>
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<td></td>
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<td></td>
<td>NS</td>
<td>15.86</td>
<td>0.0018</td>
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<td></td>
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<tr>
<td></td>
<td>CR*NS</td>
<td>1.59</td>
<td>0.2317</td>
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<tr>
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<td>CR²</td>
<td>34.26</td>
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<td></td>
<td>NS²</td>
<td>6.75</td>
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<td>Lack of Fit</td>
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<td>0.1081</td>
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</table>

The degree of precision and fitness of the developed models are checked graphically by plotting the predicted models against the actual experimental data as shown in Fig 1a and Fig 1b and they are found to align along the straight with very high degree of fitness. Therefore, the developed models are appropriate and applicable for predicting the static and dynamic MOE of RCR incorporating NS.

\[ E_C = 29.34 + 1.92CR + 16.85NS - 0.19CR \times NS - 0.23CR^2 - 11.68NS^2 + 0.067CR \times NS^2 + 0.006CR^3 + 2.05NS^3 \]  (3a)

\[ E_D = 56.929 + 0.189CR + 2.388NS - 0.033CR^2 - 1.474NS^2 \]  (3b)

where \( E_C \) is the static MOE in GPa, \( E_D \) is the dynamic MOE in GPa, CR is crumb rubber in %, NS is nanosilica in %.

![Predicted vs. Actual](image1)

a) Predicted versus actual plots for Static MOE model

![Predicted vs. Actual](image2)

b) Predicted versus actual plots for Dynamic MOE model

Fig 1: Predicted versus actual plots
3.2 Effect of Crumb Rubber and Nano Silica on the Static Modulus of Elasticity of RCR

The effect of CR and NS on the static MOE of RCR is shown in Fig 2. The static MOE of RCR decreases with partial replacement of fine aggregate with CR above 10%. This is attributed to higher deformation and lower elastic modulus of CR compared to fine aggregate it partially replaced [22, 23]. Therefore CR increases the flexibility of RCR. As seen incorporation of 10% CR increases the static MOE of RCR due to the increased consistency of RCR and higher compaction pressure used which reduces the effect of increased porosity caused by crumb rubber which leads to higher strength and consequently increased MOE. The addition of nano silica (NS) up to 2% increases the MOE of RCR for all CR replacement ratios. These findings were in agreement with results by Amin and Abu el-Hassan [24]. This increase is due to the ability of NS to react with Ca(OH)\textsubscript{2} from cement hydration to produce more calcium-silicate-hydrate which increases strength and consequently MOE. It is also due to the pore filling ability of NS, making the RCR denser, with microstructure and densified ITZ between CR-cement paste and aggregate-cement paste [25, 26].

![Fig 2: Relationship between CR, NS and static MOE of RCR](image)

3.3 Effect of Crumb Rubber and Nano Silica on the Dynamic Modulus of Elasticity of RCR

The effect of CR and NS on the dynamic MOE of RCR is shown in Fig 3. The DMOE of RCR decreases as the percentage of the crumb rubber replacement to fine aggregates increases. This decreasing is attributed to many factors such as lower specific gravity of crumb rubber compared to fine aggregate which leads to reduction in the density of the hardened RCR, and consequently, it’s DMOE. Additionally, the porosity of the hardened RCR, leads to increasing the path length through which ultrasonic wave travels, thereby decreasing the UPV, and consequently DMOE [22]. The addition of nano silica has no much effect on its DMOE as shown in Figure 3. For 1% nano silica addition slightly increased the DMOE of RCR.

![Fig 3: Relationship between CR, NS and Dynamic MOE of RCR](image)

3.4 Relationship between Static and Dynamic Modulus of Elasticity of RCR

The relationship between static (E\textsubscript{C}) and dynamic (E\textsubscript{D}) MOE of RCR is presented in Eqn 5 and Figure 4. As seen a good correlation exists between them with 60% degree of determination, and they are directly proportional to each other.

\[
E_{D} = 25.076 \times \ln(E_{C}) - 33.
\]  

(4)

4. CONCLUSIONS

In this study, based on the experimental work and analysis carried out, the following conclusions can be drawn
- The static and dynamic MOE of RCR decreases with increase in percentage replacement of fine aggregate by CR. Therefore CR increases the ductility and flexibility of RCR making it more suitable for use in the pavement.
- The addition of up to 2% NS increases by weight of cementitious materials increases both static and dynamic MOE of RCR therefore making RCR more stiff and rigid.
- From the results of RSM analysis, cubic and quadratic model types were suitable for predicting the static and dynamic MOE of RCR respectively, using CR and NS as the variables.
Fig 4: Relationship between static and dynamic MOE of RCR

5. ACKNOWLEDGEMENTS

The authors would like to thank Universiti Teknologi PETRONAS Malaysia for granting the project under code YUTP 0153AA – H30.

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