METHOD TO ESTIMATE THE SYSTEM PROBABILITY OF FAILURE FOR SLOPE STABILITY ANALYSIS

*Chollada Kanjanakul1 and Tanan Chub-Uppakarn2

1 Department of Civil Engineering, College of Industrial Technology and Management, Rajamangala University of Technology Srivijaya, Thailand; 2 Prince of Songkla University, Thailand

*Corresponding Author, Received: 21 Dec. 2017, Revised: 15 Jan. 2018, Accepted: 15 Feb. 2018

ABSTRACT: Uncertainty in soil slope is one of the problems in slope stability analysis because natural soil slope is heterogeneous that is difficult to predict time and location of the failure. This problem leads that only conventional approaches do not take into account many risks related to slope safety. The optimization of this problem is carried out using probabilistic slope stability analysis. This paper described a method to calculate the probability of failure of slope stability analysis. The system probability of failure is defined as the complement of the sum of the probability of failure corresponding to sliding failure. Furthermore, this paper shows time-steps in the developments of factors of safety in soil slope and perform natural slope stability analyses by probabilistic approach on a hill range. Sensitivity result indicated that the effective angle of internal friction (φ) and the angle of soil slope (β) are most significant parameters and choose to be random variable parameters. Probabilistic for soil slope failure is calculated using conventional design equations, mean and coefficient of variation values for the random variable parameters as input. GeoStudio’s program (SEEP/W and SLOPE/W application) have been employed to describe the process of rainfall infiltration under positive and negative pore-water pressures and slope stability analyses, respectively. Bishop’s simplified method was used in conjunction with Monte Carlo Simulation to determine in term of the probability of failure (Pf). This approach is to prove the best confidence result in slope stability analysis.

Keywords: Slope stability analysis, Rainfall-induced landslides, Random variable, Probability of failure

1. INTRODUCTION

Rainfall-induced slope failure is a natural disaster and geotechnical problem in a tropical area. The effects of a slope stability depend on many factors such as hydrology (e.g., rainfall characteristics), soil properties (e.g., unit weight, angle of friction, cohesion of soil), hydrogeology (e.g., hydraulic conductivity, moisture content, groundwater table), and others such as vegetation cover [1]. To mitigate the problem, engineers have constantly tried to find better methods of warning systems. For example, Casagli N., Dapporto S., Ibsen M. L., Tofani V., and Vannocci P. [2] analyze two landslides in Northern Tuscany by modeling the process of rainwater infiltration. For both sites, result from morphometric and geotechnical analyses was used as a direct input to the numerical modeling. A Finite element analysis was used to model the fluctuations in pore water pressure during the storm. This was then followed by the application of the limit equilibrium method. From this methodology, a trend for the factor of safety was produced for both landslide sites. This result indicates that the most critical time step for failure was a few hours following the rainfall peak, while Bathrellos G. D., Kalivas D. P., and Skilodinou H.D. [3] presented the production of landslide susceptibility maps in the mountainous part of the Trikala Prefecture in Thessaly, Central Greece based on GIS techniques using two different models of combining the instability factors and estimation of overall landslide susceptibility, namely: the Weight Factor Model (WeF), which is a statistical method, and the Multiple Factor Model (MuF) that is a logical method. The produced maps were classified into four zones: Low, Moderate, High and Very High susceptible zones and validated using the other half number of the landslide events of the area. Evaluation of the results is optimized through a Landslide Models Indicator (La.M.I.).

Landslides are most common in Thailand, causing high damages and costs. The rate of water infiltration plays important roles in triggering landslides. Antecedent precipitation index (API) in soil engineering is an index of static soil moisture stored in the soil mass when the soil slope starts to become unstable and is used as an indicator of the degree of saturation which relates to the gravitational force and the soil resisting forces in slope stability problem. Many studies have been conducted to investigate the relationship between API and the rate of water infiltration. Chollada K., Tanan C., and Tanit C. [4] studied and analyzed the influence of unsaturated slope stability with regard to the outcome of calculated APIcr versus the actual APIcr value in an occurred landslide disaster on a
hill range in one southern part of Thailand (Khao Luang range in Nakhon Si Thammarat province) in order to verify the use of APIcr for an early warning indicator if and when a future unusual rainfall happens in the area.

Furthermore, spatial variability of soil properties has often ignored in geotechnical design. This may lead to decrease reliability and unsafe in slope stability analysis because in slopes with the same safety factor value may exhibit different risk levels depending on the variability of the soil properties. In other words, a factor of safety is not a consistent measure of risk [5]. To solve the problem from this type of disasters, reliability analysis and landslide early warning systems have long been under development.

This paper presents a method to calculate the system probability of failure of a soil slope in the tropical region. Probabilistic slope stability analysis was performed to quantify and study the effects of uncertainty due to the variability of soil properties on a natural slope. Moreover, quantifying uncertainty was used to evaluate the reliability of landslide warning system on a natural slope.

2. SLOPE GEOMETRY

The model for this case is shown in Figure 1, the slope geometry in this study were based on typical residual soils in the tropical region and the works by Chollada K. [6]. At the top, there is a heterogeneous 1-5 m unsaturated topsoil layer overlaying a 1 m thick decomposite rock on top of a 9-26 m thick bedrock. Topsoil strength properties obtained from Phoon K. K., and Kulhawy F. H. [7], [8] are 1-35 kPa, the effective soil cohesion; 20°-30°, the effective angle of internal friction; 10°-20°, the angle of friction with respect to matric suction in unsaturated soils; and 13-20 kN/m³, the soil unit weight.

3. METHODOLOGY

This methodology can be separated into four stages (Table 1): (i) Preparation of slope models for slope stability analysis; (ii) Calculation of factors of safety of the slope by SEEP/W and SLOPE/W; (iii) Calculation of antecedent precipitation index (API) from stability analysis results; (iv) Sensitivity analysis for natural slope; (v) The use of sensitivity analysis results to calculate probability of failure (Pf).

3.1 Preparation of Slope Models for Slope Stability Analysis

The mathematical model for this case is shown in Figure 1. A slope height of 34 m and a slope angle of 20°-50°. Initial depth of groundwater table was stated at the top of a bedrock. In the finite element analysis, the slope profile was divided into meshes of equal quadrilateral elements with a total number of 1479 elements. Boundary conditions utilized for the transient seepage analysis are: Zero flux for the lower and the upper vertical bed boundaries (there is no seepage through the base of the soil slope); A unit gradient (i) for the lower vertical boundary of the section; and a rainfall intensity Ir for the upper horizontal boundary.

3.2 Calculation of Factors of Safety to the Slope by SEEP/W and SLOPE/W

Factors of safety for slope stability analysis can be performed in two steps; the use water infiltration rates as surface boundary conditions to model fluctuations in pore-water pressure by SEEP/W, positive and negative pore-water pressures obtained were then used in SLOPE/W employing limit equilibrium method to calculate the factor of safety of the slope. Unsaturated flow is based on modified Darcy’s law; the differential equation for a two-dimensional transient water flow utilized in SEEP/W model is as follow:

\[
\frac{\partial}{\partial x} (k_x \frac{\partial H}{\partial x}) + \frac{\partial}{\partial y} (k_y \frac{\partial H}{\partial y}) + Q = \frac{\partial \theta}{\partial t} \tag{1}
\]

Where \( H \) is the total head, \( k_x \) is hydraulic conductivity in the x-direction, \( k_y \) is hydraulic conductivity in the y-direction, \( Q \) is the applied boundary flux, \( \frac{\partial \theta}{\partial t} \) is the volumetric water content, and \( t \) is the time.
The fluctuations in pore water pressure during a rainfall obtained from the transient seepage analysis using SEEP/W were exported to SLOPE/W to compute the slope stability. Slope stability studies are commonly based on calculations of a factor of safety considering a failure surface. Expressing by F.S., Bishop’s simplified method (Eq. (2)) was adopted in the slope stability analysis. This method is capable of calculating F.S. with accuracy close to other more rigorous methods [9].

\[
F.S. = \frac{c' + (\sigma_n - u_a) \tan \phi + (u_a - u_w) \tan \phi_b}{\gamma H \sin \beta}
\]

(2)

Where \(c'\) is the effective cohesion, \(\phi\) is the effective angle of internal friction, \((\sigma_n - u_a)\) is the effective normal stress on the plane of failure, \((u_a - u_w)\) is matric suction on the plane of failure, \(\phi_b\) is the angle of friction with respect to matric suction in the unsaturated soil, \(\gamma\) is the unit weight of the soil, \(H\) is the thickness of the soil slope, and \(\beta\) is the slope angle.

### 3.3 Calculation of Antecedent Precipitation Index (API) from Stability Analysis Results

The Antecedent Precipitation Index (API) does not only describe the maximum amount of water that a soil slope can maintain in its layer without unstable but also used to trigger a warning to the landslide risk area. An equation to calculate API from direct precipitation measures [9], as follows:

\[
API(t) = (API(t-1) \cdot K(t-1)) + P(t)
\]

(3)

where \(API(t)\) is API of a current period (mm), \(API(t-1)\) is API of the previous period (mm), \(P(t)\) is precipitation of the current period (mm), and \(K(t-1)\) is a recession constant for the previous period.

Choudhury [11] presented the estimate of \(K(t-1)\) shown in Eq. (4)

\[
K(t-1) = \exp(-Et/W)
\]

(4)

where \(Et\) is evaporation at short shrift, and \(W\) is the soil moisture at evaporation time.
Three further parameters ($V_v$, $V_t$, and $V_w$) needed in the API calculation can be described as phase relationships for the soil

The phase ratios are: porosity $\eta = \frac{V_v}{V_T}$, volumetric water content $\theta = \frac{V_w}{V_T}$, and degree of saturation $S_r = \frac{V_w}{V_v}$.

The critical API is estimated using these relations and is used to trigger a warning to the landslide risk area. To determine this critical API, we need the maximum amount of water or the critical moisture content, and the wetting front thickness in the soil. Then the critical API is estimated as

$$API_{cr} = \eta S_{cr} T_{cr}$$

where $S_{cr}$ is the critical degree of saturation, and $T_{cr}$ is the critical thickness of the soil layer.

### 3.4 Sensitivity Analysis for Natural Slope

As mentioned earlier, the slope geometry data from [6] were used as inputs in slope stability analysis. The sensitivity analysis was performed to investigate parameters on a rainfall-induced landslide. In addition, the method was used to find the most effective parameters on the stability of slopes that is characterized by a factor of safety, API and time when slope become unstable based on limit equilibrium analysis method. In this analysis, the geometry and shear strength parameters were varied that are summarized in Table 2.

**Table 2 Sensitivity analysis (Adapted from [6])**

<table>
<thead>
<tr>
<th>Series</th>
<th>$\beta$ (°)</th>
<th>$I_r$ (m m/hr)</th>
<th>T (m)</th>
<th>$c'$ (kPa)</th>
<th>$\gamma$ (kN/m$^3$)</th>
<th>$\phi$ (°)</th>
<th>$\phi_b$ (°)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>20-50</td>
<td>6-36</td>
<td>1-5</td>
<td>14.37</td>
<td>16.8</td>
<td>24</td>
<td>10</td>
</tr>
<tr>
<td>B</td>
<td>26</td>
<td>6-36</td>
<td>3</td>
<td>13-20</td>
<td>20-30</td>
<td>10-20</td>
<td></td>
</tr>
</tbody>
</table>

**Table 2. Series A (Geometry):** The soil thickness and slope angles for the scenarios were chosen based on GIS data of the study area. The slope angles range within 20-50° and the soil thickness within 1-5 m. The rainfall intensities 6-36 mm/hr used in the sensitivity analysis were adopted from the intensity-duration-frequency (IDF) curve for the southern part of Thailand.

**Table 2. Series B (Soil strength):** $c'$ (1-35 kPa), $\gamma$ (13-20 kN/m$^3$), $\phi$ (10-20°) ranges were chosen based on approximate guidelines on soil property variability [6], and the range of $\phi$ (20-30°) was chosen based on prior published descriptive properties of soil [12].

### 3.5 The Use of Sensitivity Analysis Results to Calculate Probability of Failure ($P_f$)

From Sensitivity result, the authors can find the most significant parameters and choose to be random variable parameters. The probability of failure ($P_f$) for geotechnical problems can be determined by using a variety of methods [13]. First order second moment or point estimate methods can be used when the distribution of the Factor of safety (F.S) is approximately normal while the Monte Carlo method is generally used to determine the distribution of the F.S and $P_f$ [14].

#### 3.5.1 Quantifying uncertainty in soil properties

[15] described that engineers can be adjusted for best estimate and a measure of uncertainty in the best estimate.

- Mean ($\mu_X$) is a statistical measure of normal distribution and measure of unimodal pattern data. That data collected from a population with a constant standard deviation.

$$\mu_X = \frac{1}{N} \sum_{i=1}^{N} x_i \mu_X$$

- Standard deviation ($\sigma_X$) is the square root of the variance which is used to explain the deviation of population

$$\sigma_X = \frac{1}{N-1} \sum_{i=1}^{N} (X_i - \mu_X)^2$$

- The coefficient of variation (COV) is consideration ratio of standard deviation by mean for compare standard deviation data more than two values.
\[ COV_x = \frac{\sigma_x}{\mu_x} \]  

(8)

The statistic values of random variable parameters can be explained by mean \( \mu_x \), standard deviation \( \sigma_x \) and Coefficient of variation \( COV_x \). Random variable parameters are generated base on the assumption of the probability density function (PDF) and the performance function is estimated for each generated set. The process is repeated several times to calculate the probability of failure (Pf).

Chollada [6] used Monte Carlo simulation to develop a probabilistic design method for rainfall-induced slope. Monte Carlo simulation is a method used to obtain the probability distribution of random variables given the probability distribution of a set of random variables (Fig 2). The probability density function for each of these soil variables must be specified. Then, the uncertainty in slope stability was quantified by evaluating the probability of failure. They were calculated using the safety factor probability distribution.

The method used to compute Pf can be summarized in the following steps:

1) Determine the geometry and soil strength parameters of the slope: The height of soil slope (D), the soil thickness (t), Slope angles (\( \beta \)), The effective cohesion (\( c' \)), The unit weight of the soil (\( \gamma \)), The effective angle of internal friction (\( \Phi \)), The angle of friction with respect to matric suction in the unsaturated soil (\( \beta \)).

2) Random variable (RV) parameters can be defined as a variable and explained in a statistically significant function by a mean (\( \mu \)) and coefficient of variation (COV).

3) Compute the stability of soil slope by Bishop’s simplified method.

4) Compute n time of F.S. start from i=1 until i=n (F.S.1, F.S.2, ..., F.S.n) that the result can be expressed as probability density function (pdf) of F.S.

The probability of failure \( P_f \) can be calculated by Monte Carlo Simulation and calculated from F.S. i amount 2,000 times. In the same number of F.S. i<1 divided by the number of 10,000 times will be Probability of failure \( P_f \). (Eq.9)

\[ P_f = \frac{F.S. < 1}{2,000} \]  

(9)

4. RESULTS AND DISCUSSION

As earlier described, a typical geometry of a heterogeneous soil slope was earlier shown in Fig 1.

Rainfall intensities 6-36 mm/h from the Thailand’s intensity-duration-frequency curve were used in SEEP/W and SLOPE/W programs to compute the factors of safety in the soil slope.

4.1 Slope Stability

Stability analysis results computed using SLOPE/W were plotted showing six separate time-steps in the developments of factors of safety (Figure 3). At case study model, rainfall infiltration process in the soil mass led to the decreasing of factor of safety and soil strength. In Figure 3, at the beginning factor of safety varied from 1.16 to 1.07 (step 1 at 0 h, to step 3 at 23 h). At 100 h, the increase in rainfall intensity lowered the factor of safety down from 1.16 to 0.91 (step 1 to step 6 in Figure 3). Thus, the factor of safety declined to the point of the slope became unstable at approx. 53 hours after the initiation of the rainfall. The corresponding Pf was computed to be 0.558. This Pf result agrees well with the actual F.S. value that cumulative rainfall starting from the first day of this rain spell up to 53 hours, the time that landslides occurs.

4.2 Sensitivity Analysis

Sensitivity analysis results computed using SEEP/W and SLOPE/W were written showing separate series table: (i) The effects on time (hour) when slope become unstable of various geometry parameters table (Series A); (ii) The effects on time (hour) when slope become unstable of various soil strength parameters table (Series B).

From equation 9 Sensitivity is expressed by a dimension less index \( I' \), which is calculated as the ratio between the relative changes of a parameter. The sensitivity index as defined by Lenhart T., Eckhardt K., Fohrer N., and Frede H. G. [17] is calculated and analyses the effect of input parameter on the factor of safety or stability of internal dump slope.

\[ I' = \frac{(y'_2 - y'_1)}{y'_0} \]  

\[ \frac{2\Delta x}{x_0} \]  

(10)

According to sensitivity analysis series A, B a dimension less index can be calculated as shown in Table 3.

4.2.1 The influences of geometry parameters

The effect of slope geometry is evaluated in terms of slope angle (\( \beta \)) and slope thickness (T).
Fig. 2 Probabilistic slope stability analysis process (Adapted from [6])

Table 3 (Series A) shows the time when slope became unstable, consistently increased with soil thickness and reached 30 hours at 5 m thickness. However, increasing the slope angle from 20 to 50 decreased the time to 20 hours. The water reservoir capacity increases with thickness and makes the slope more stable, and also a reduced slope angle naturally contributes to slope stability.

Table 3 (Series B) shows the time over ranges of soil strength parameters: effective cohesion ($c'$), effective angle of internal friction ($\phi$), the angle of friction with respect to matric suction in the unsaturated soil ($\phi^b$) and density of soil ($\gamma$). The time increased sharply between 42 and 71 over the effective cohesion range with 35 kPa width; this was the largest effect size observed. The $\phi^b$ range with width 12 caused 25 hours change in API, the $\gamma$ range with width 7 caused 14 hours change in time, and $\phi$ range from 20 to 30 correspond to 25 hours change in time. Soil density had the smallest effect on time. In series B, It can be seen that shear strength of soil increased with $\phi$, $\phi^b$, $c'$, $\gamma$ and makes the slope more stable.

From the influences of slope geometry and soil strength parameter graphs in sections 4.2.1 and 4.2.2 and summary of sensitivity analysis (slope geometry and soil strength parameters) with sensitivity indexes (Table 3). Dimensionless index $I'$, is the best value to arrange in order of parameters. The increase in dimensionless index value increasing the significant of parameter.

The results indicated that the angle of soil slope ($\beta$) is most significant in slope geometry parameters with -0.519 of dimension less index, $I'$ and the effective angle of internal friction ($\phi$) is most significant in soil strength parameters with 1.19 of $I'$ ($\beta$ and $\phi$ had the highest effect on time when their range was small change) and choose to be random variable parameters.

Fig. 3 Time-step versus factor of safety during the rainfall

4.2.2 The influences of soil strength parameters

Table 3 (Series B) shows the time over ranges of soil strength parameters: effective cohesion ($c'$), effective angle of internal friction ($\phi$), the angle of friction with respect to matric suction in the unsaturated soil ($\phi^b$) and density of soil ($\gamma$). The time increased sharply between 42 and 71 over the effective cohesion range with 35 kPa width; this was the largest effect size observed. The $\phi^b$ range with width 12 caused 25 hours change in API, the $\gamma$ range with width 7 caused 14 hours change in time, and $\phi$ range from 20 to 30 correspond to 25 hours change in time. Soil density had the smallest effect on time. In series B, It can be seen that shear strength of soil increased with $\phi$, $\phi^b$, $c'$, $\gamma$ and makes the slope more stable.

From the influences of slope geometry and soil strength parameter graphs in sections 4.2.1 and 4.2.2 and summary of sensitivity analysis (slope geometry and soil strength parameters) with sensitivity indexes (Table 3). Dimensionless index $I'$, is the best value to arrange in order of parameters. The increase in dimensionless index value increasing the significant of parameter.

The results indicated that the angle of soil slope ($\beta$) is most significant in slope geometry parameters with -0.519 of dimension less index, $I'$ and the effective angle of internal friction ($\phi$) is most significant in soil strength parameters with 1.19 of $I'$ ($\beta$ and $\phi$ had the highest effect on time when their range was small change) and choose to be random variable parameters.
Table 3 The relationship between slope geometry, soil strength parameters and the time when slope became unstable

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Time (Hour)</th>
<th>Parameter</th>
<th>$I'$</th>
<th>sequence</th>
</tr>
</thead>
<tbody>
<tr>
<td>T (m, Series A)</td>
<td>35-65</td>
<td>1-5</td>
<td>0.450</td>
<td>5</td>
</tr>
<tr>
<td>$\beta$ (°, Series A)</td>
<td>55-35</td>
<td>20-50</td>
<td>-0.519</td>
<td>4</td>
</tr>
<tr>
<td>$\phi_b$ (°, Series B)</td>
<td>50-75</td>
<td>10-20</td>
<td>0.6</td>
<td>3</td>
</tr>
<tr>
<td>$\phi$ (°, Series B)</td>
<td>40-65</td>
<td>20-30</td>
<td>1.19</td>
<td>1</td>
</tr>
<tr>
<td>$\gamma$ (kN/m$^3$, Series B)</td>
<td>46-60</td>
<td>13-20</td>
<td>0.63</td>
<td>2</td>
</tr>
<tr>
<td>$\epsilon'$ (Series B)</td>
<td>42-71</td>
<td>1-35</td>
<td>0.27</td>
<td>6</td>
</tr>
</tbody>
</table>

4.3 Probabilistic Slope Stability Analysis

Figure 4 shows probabilistic slope stability analysis from Geostudio’s program. In case study, the effective angle of internal friction ($\phi$) is selected to be random variable with $\mu_{\phi}$ (mean) = 25° and COV$_{\phi}$ (coefficient of variation) = 3 [7], [8]. The result shows Probability Density Function, PDF when the author computes the distribution of factors of safety by Bishop’s Simplified method with Monte Carlo simulation amount of 2,000 times. The result can be expressed in histogram in the function of mean of factors of safety, $\mu_{F.S.}$ equal 0.558. Stability analysis results computed using SLOPE/W at approx. 53 h after the initiation of the rainfall the factor of safety declined to the point the slope became unstable. The probability of failure was computed to be 0.558.

5. CONCLUSION

In soil slope analyses, factor of safety at six time-steps were evaluated. Rainfall infiltrates through the soil increases the soil moisture content. The rainfall intensity leading to reduction of matric suction and the factor of safety. The unsaturated soil started from its natural volumetric water content. From slope stability analyses using SEEP/W and SLOPE/W saturation in the layer rose until the colluvium became unstable at 53 h, the increase in rainfall intensity lowered the factor of safety down from 1.16 to 0.98. Sensitivity analysis result showed that the effective angle of internal friction ($\phi$) and the angle of soil slope ($\beta$) are the most significant parameters and chosen to be random variable parameters. A method for calculated the system probability of failure and reliability for soil slope having a slope height of 34 m and a slope angle of 26° (mean of slope angle) has been described. The probability of failure is calculated using the mean factor of safety computed using Bishop’s simplified method and statistical data (mean and coefficient of variation) of random variable parameters.

6. ACKNOWLEDGEMENTS

We are very grateful to the Southern Natural Disaster Research Center of the Prince of Songkla University (PSU), Thailand, in providing relevant geological details and geographical information of the landslide disaster area studied in this work. The Thailand Royal Irrigation Department is sincerely
acknowledged for providing the rainfall data in its vicinity.

7. REFERENCES


Copyright © Int. J. of GEOMATE. All rights reserved, including the making of copies unless permission is obtained from the copyright proprietors.