AN EMPIRICAL MODEL FOR PREDICTING HYDRAULIC CONDUCTIVITY OF MINE TAILINGS

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ABSTRACT: To evaluate the applicability of mine tailings as embankment materials, its geotechnical characteristics have to be established. Standard ASTM procedures are performed to determine the physical properties and hydraulic conductivity of mine tailings from three (3) different mining sites in the Philippines. Falling head permeability tests are performed on reconstituted samples compacted at varying void ratio and fine content. The hydraulic conductivity, k of tailings falls within the range of $10^{-3}$ to $10^{-6}$ cm/sec. Data obtained from these tests are used to develop an empirical model for predicting the hydraulic conductivity of mine tailings under fully saturated conditions as a function of void ratio and fine contents. The formulated model is a useful tool to estimate the hydraulic conductivity of non-plastic fine tailings, a parameter needed in seepage analysis, when these waste materials are used as embankment materials in the construction of tailing dams.

Keywords: Mine Tailings, Geotechnical Characteristics, Hydraulic Conductivity

1. INTRODUCTION

Despite the economic benefits, mining operations often face strong opposition from affected communities because of its adverse environmental and social effects. The disposal of the enormous amounts of tailings regularly produced from mining operations is the most common environmental issue associated with mining activities. In an effort to protect the environment, it has been an integral part of most mining companies to undertake environmental protection measures in order to minimize if not completely eliminate negative environmental impacts. Some of these measures are the construction of tailing disposal systems. The most common storage methods used in tailing disposal are the use of dams, embankments and other types of surface impoundments [1]. In tailing dams, the mine tailings are generally deposited in slurry form. As the tailings settle and solidify, water rises to the surface from where it can be decanted. Failure of the dam, while the mine tailings are still in slurry form, can result in a debris flow that poses a serious threat to life, property and the environment.

It is important to reduce the volume of tailings in storage facilities so that the risk to the exposed population and the environment can be reduced. One possible option is to utilize tailings that do not contain deleterious components as backfill or as embankment materials in the construction of tailing dams. To evaluate the applicability of mine tailings as embankment materials, its geotechnical characteristics have to be established. In the Philippines, most studies on mine tailings have focused on its effect to the environment; no study about its geotechnical characteristics has been conducted. Considering the large volume of tailings generated and currently in storage facilities together with the increasing pressure on the mining industry to sustain stringent safety and environmental standards, this research study aims to contribute knowledge, data and insights on the geotechnical characteristics of mine tailings in the Philippines in order to determine its possible re-use as fill or embankment materials.

One of the most important geotechnical parameters which need to be established is the hydraulic conductivity because this influences the consolidation behavior and the seepage condition in the contained tailings and through the dams. In Iran, mine tailings were used as construction materials in increasing the height of the Sarcheshmeh copper tailing dam [2]. A case study was conducted to determine the geotechnical characteristics of Sarcheshmeh original mine tailings as well as the properties of hydro cyclone underflow coarse grained material through series of laboratory experiments. One output of the study was the formulation of new relation for estimating hydraulic conductivity in terms of the percentage finer than sieve #200. A laboratory investigation on the hydraulic conductivity of four tailings from hard rock mines from mining sites in the Abitibi region, in the western part of Quebec, Canada was undertaken [3]. The hydraulic conductivity of these waste materials is represented by a modified version of the Kozeny-Carman equation in which a tortuosity factor and a grain-size distribution function are included explicitly.
This study determines the physical properties and hydraulic conductivity of tailings in the Philippines. Physical properties which include grain-size distribution, Atterberg limits, specific gravity, maximum and minimum index densities, soil classification, and microfabric structures are presented. Test results from hydraulic conductivity tests using rigid-wall permeameter under falling condition are discussed. From test results, a model was formulated to predict the coefficient of hydraulic conductivity \( k \) of tailings taking into account the effect of void ratio and fine contents.

2. THE TAILING SAMPLES

Tailing samples came from three (3) mining sites in the Philippines namely concrete aggregate quarry in Cavite, gold processing plant in Davao del Norte, and gold mining site in Aroroy, Masbate. The first sample designated as TS#1 is tailing of concrete aggregate quarry. Tailings are produced from washing crushed rocks in the siltation pond through the natural process of sedimentation. The second sample (TS#2) is tailing from a gold processing plant located at Davao del Norte. Gold content is separated from mined ore by carbon-in-leached process. Wastes coming out of the plant flow to an impounding pond where tailings are stored. The third sample (TS#3) is tailing from gold mine site in Aroroy, Masbate. Wastes from the processing plant are stored in the tailing dam. The three (3) types of tailing samples were taken from old stock pile of tailings at random sampling.

3. PHYSICAL PROPERTIES

Standard ASTM procedures are performed to obtain the physical properties of tailing samples. The distribution of grain sizes of the three (3) types of tailing samples was determined using the combined method of sieve analysis and hydrometer test in accordance with ASTM D422. Fig. 1 illustrates the grain size distribution of tailing samples. Tailings from concrete aggregate quarry (TS#1) consisted primarily of fine sands with very few silts. Based on Unified Soil Classification System (USCS), TS#1 is classified as poorly graded sand with silt and is given the symbol of SP-SM. Gold mine tailings from Davao (TS#2) and Masbate (TS#3) both exhibited an almost equal distribution of fine sands and silts. TS#2 and TS#3 both have USCS classification of silty sand with symbol of SM.

![Grain-size distribution curve](image)

Table 1

<table>
<thead>
<tr>
<th>Soil constant</th>
<th>TS#1</th>
<th>TS#2</th>
<th>TS#3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Specific gravity, ( G_s )</td>
<td>2.57</td>
<td>2.72</td>
<td>2.71</td>
</tr>
<tr>
<td>Liquid Limit, LL %</td>
<td>27</td>
<td>24</td>
<td>23</td>
</tr>
<tr>
<td>Plasticity Index, PI %</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Shrinkage Limit, %</td>
<td>21</td>
<td>20</td>
<td>20</td>
</tr>
<tr>
<td>Shrinkage Ratio</td>
<td>1.47</td>
<td>1.57</td>
<td>1.66</td>
</tr>
<tr>
<td>Min. Void Ratio, ( e_{\min} )</td>
<td>0.624</td>
<td>0.680</td>
<td>0.662</td>
</tr>
<tr>
<td>Max. Void Ratio, ( e_{\max} )</td>
<td>1.024</td>
<td>1.106</td>
<td>1.089</td>
</tr>
<tr>
<td>Max. dry unit weight, ( \gamma_{d,max} ) kN/m³</td>
<td>15.56</td>
<td>17.12</td>
<td>17.72</td>
</tr>
<tr>
<td>Optimum moisture content, ( w_{opt} ) %</td>
<td>13.49</td>
<td>17.28</td>
<td>12.82</td>
</tr>
</tbody>
</table>

4. MICROFABRIC ASSESSMENT

The study of tailing’s micro fabric was undertaken through the use of scanning electron microscope (SEM). Fig. 2 shows the micrographs of the 3 tailing samples. The micro fabric of TS#1 comprised of granular particle arrangement with clean contacts. The structure comprised of a combination of rounded and sub-angular grains with larger sizes and few silt-size grains. The micro fabric of TS#2 consisted mainly of well-rounded and elongated granular particle arrangement of smaller sizes with more silt grains. A combination of extremely large angular grains and abundant silt grains formed the micro fabric of
TS#3. The micrograph clearly shows the irregular shapes and rough surface indicating a clothed silt grains contacts. Similar to TS#2, which is classified as silty sand, silt grains dominate the fabric of this sample.

5. HYDRAULIC CONDUCTIVITY

The hydraulic conductivity of three types of tailings was evaluated using rigid-wall permeameter with internal diameter of 6.33 cm and sample height limited to 14 cm. The permeability tests were performed under a falling head condition with a minimum hydraulic gradient of 4. Reconstituted specimens are prepared with varying void ratio, \( e \) and fine contents, \( F \) to investigate the effect of void size on the hydraulic conductivity, \( k \) of tailings. The hydraulic conductivity, \( k \) was calculated using the Darcy’s law. A total of 98 test runs were conducted and the measured values of \( k \) for different void ratios and fine contents are presented in Fig. 3.

Fig. 2 Micrographs of tailing samples at 500X magnification

Fig. 3 Measured \( k \) vs. void ratio, \( e \) (a) TS#1 (b) TS#2 (c) TS#3
As can be seen from the results, \( k \) values fall within the range of \( 10^{-3} \) to \( 10^{6} \) cm/sec. These tailings are considered to have low to very low permeability typical of silty sand or silty clay. The measured \( k \) values are within the range as those reported for similar tailings in the literature: \( 10^{-4} \) to \( 10^{-6} \) cm/sec for homogenized tailings [3], \( 10^{-5} \) cm/sec for gold tailings [4], \( 10^{-4} \) to \( 10^{-6} \) cm/sec for copper tailings [5].

5.1 Formulation of Hydraulic Conductivity Model

The hydraulic conductivity also called coefficient of permeability, of soils, is influenced by various physical factors. Hydraulic conductivity of soils is significantly affected by the void size which is related to particle size of the soil [6]. Several researchers have formulated equations to predict hydraulic conductivity of various types of soils and tailings. Hazen’s equation was intended for uniform loose sand and had been used also to estimate hydraulic conductivity of tailings [7]. It is usually written in this form:

\[
k = c_2 D_{10}^2
\]

where \( k \) is given in cm/sec, \( D_{10} \) is the grain diameter in cm with ten percent (10%) passing, and \( c_2 \) is considered a material constant. As suggested by various authors [1], [8], a value of 100 may be adopted for \( c_2 \). However, it should be noted that the material constant \( c_2 \) can vary between 60 and 150 depending upon grain-size distribution [9].

Another formula, as shown in Eq. (2), was specifically developed by Bates and Wayment for tailings at the U.S. Bureau of Mines [10].

\[
k = \exp \left[ x_1 + x_2 \ln (eD_{10}) + x_3 \ln (e) \ln (C_u) + x_4 (eC_u) + x_5 (D_{10}D_{50}) \right]
\]

The \( k \) unit is in inches per hour and the following values for the constants have been proposed: \( x_1 = 11.02, x_2 = 2.912, x_3 = -0.085, x_4 = 0.194, x_5 = -56.49 \). This equation was based from 100 infiltration tests results; range of void ratios was between 0.52 and 1.08, \( D_{10} \) values between 0.003mm and 0.105mm, \( D_{50} \) values between 0.060mm and 0.24mm, and \( C_u \) values between 2 and 22.

Aubertin, et al. modified the Kozeny-Carman equation to develop a model that can be used to predict \( k \) of homogenized hard rock mine tailings [3]. It is given in this form:

\[
k = c \frac{\gamma_w D_{10}^2 C_{1/3}^5}{\mu (1 + e)}
\]

in which \( c \) has an average value of 0.004, \( e \) is the void ratio, \( \gamma_w \) is the unit weight of water at room temperature (9.8 KN/m\(^3\)) and \( \mu \) is the fluid viscosity (9.8 x 10\(^{-6}\) N-s/cm\(^2\)). Both parameters are temperature dependent.

Shamsai et al. formulated the relation between \( k \) value of Sarcheshmeh copper mine tailings and the void ratio \( e \) [2], and can be stated as follows:

\[
k = 0.09 \times 10^{-0.08 P_{200}} \left( \frac{e^{2.8}}{e + 1} \right)
\]

where \( P_{200} \) is the percentage finer than sieve #200 and \( e \) is the void ratio. The parameter \( P_{200} \) is considered to replace the grain size. It has a limitation to be over 50%.

In this study, the fine content \( F \) was considered as a new parameter in lieu of the grain size, similar to the work of Shamsai [2], but allowing wider range of fine contents; 0% or no fine contents \( (F=0) \), 10% \( (F=10) \), 30% \( (F=30) \), 50% \( (F=50) \) and 70% \( (F=70) \) passing the #200 sieve. The fine contents \( F \) of tailings is represented by the percentage passing the #200 sieve. It is assumed in this study that the reconstituted samples are isotropic and the experimentation determines the flow of water on the saturated sample in the vertical direction.

The measured \( k \) is a dependent variable which is a function of \( e \) and \( F \). The statistical relationship between the dependent variable, \( k \) and independent variables, \( e \) and \( F \) are determined empirically through multiple regression analysis. The regression model is expressed as:

\[
\ln k = b_1 + b_2 x_2 + b_3 x_3
\]

where \( b_1, b_2, b_3 \) are regression coefficients that are estimated from the experimental data. \( x_2 \) and \( x_3 \) are the independent variables \( e \) and \( F \), respectively. Since \( k \) is a nonlinear function as it is exponentially related to \( e \) and \( F \), the logarithmic function of \( k \) is used to convert it into a linear function. The strength of linearity underlying a regression equation is measured by the corresponding correlation coefficients; a high correlation coefficient (i.e. close to ±1.0) implies the existence of a strong linear relationship between the variables, whereas a low correlation (close to zero) would mean the lack of linear relationship.

The output of regression analysis is a model that predicts the hydraulic conductivity of the three
tailing samples (TS#1, TS#2, and TS#3) which is in the form:

\[ k = c f_e f_d \]  (6)

The function \( f_e \) reflects the influence of void ratio while \( f_d \) function represents the effect of fine contents. \( f_e \) and \( f_d \) are in the form:

\[ f_e = \frac{e^n}{1 + e} \]  (7)
\[ f_d = \exp(c_2 F) \]  (8)

where \( c, c_2 \) and \( n \) are material constants for a specific type of tailing, \( e \) is the void ratio and \( F \) is the fine content. The relation of \( k \) to void ratio and fine contents in the form of Eq. (6) is similar to existing empirical equations used for soils [7] and tailings [2],[3],[10], but these equations contain some material constants which are unique for certain type of soils and tailings, and may not be applicable to tailings in this study.

From test results, the hydraulic conductivity of tailing samples as a function of void ratio \( e \) and percentage of fine content passing the #200 sieve (e.g. \( F=10 \) for 10% fine contents) can be expressed assuming the following 3-parameter exponential relationship:

\[ k = c \exp(-c_2 F) \left( \frac{e^n}{1 + e} \right) \]  (9)

For the three (3) tailing samples tested in this study, the following table summarizes the parameters \( c, c_2 \) and \( n \) obtained by multiple regression, together with the corresponding coefficient of determination \( R^2 \). The high values of coefficient of determination indicate that the model fits the data well.

Table 2 Model coefficients obtained from regression analysis.

<table>
<thead>
<tr>
<th>Sample</th>
<th>( c ) (cm/sec)</th>
<th>( c_2 )</th>
<th>( n )</th>
<th>( R^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>TS#1</td>
<td>( 5.81 \times 10^{-3} )</td>
<td>0.047</td>
<td>6.69</td>
<td>0.989</td>
</tr>
<tr>
<td>TS#2</td>
<td>( 3.52 \times 10^{-3} )</td>
<td>0.043</td>
<td>9.74</td>
<td>0.982</td>
</tr>
<tr>
<td>TS#3</td>
<td>( 5.68 \times 10^{-3} )</td>
<td>0.045</td>
<td>7.01</td>
<td>0.988</td>
</tr>
</tbody>
</table>

To further validate the capabilities of the proposed model, the \( k \) values calculated using Eq. (9) was compared with the measured \( k \) values. Fig. 4 shows the corresponding relationship of predicted \( k \) with the measured \( k \). From this plot, it can be demonstrated that the proposed model (Eq. 9) provides a good predictor of \( k \) for a given value of fine contents, \( F \) and void ratio, \( e \).
The calculated values of $k$ using Eq. (9) were normalized by $f_e$ (Eq. (8)) in order to evaluate the effect of void ratio to hydraulic conductivity $k$ of tailings. As can be seen from Fig. 5, the three tailing samples exhibited the same trend; the $k$ values exponentially increase as the void ratio increases. The $k$ value as a function of void ratio of TS#2 is slightly lower than the $k$ values of the other 2 samples. The elongated grain particle of smaller sizes and more silt grains found in TS#2 results to sample with less inter-granular void spaces and consequently lower $k$. On the other hand, TS#1 which has larger grains with rounded and sub-angular shapes results to more inter-granular void spaces and higher $k$ than TS#2. TS#3 which has more silt grains like TS#2 has $k$ value similar to TS#1 rather than TS#2 because its grading characteristic is gap-graded comprising of extremely large and extremely small angular grains which lead to lose packing.

Fig. 5 Predicted $k$ of mine tailings as a function of void ratio

To quantify the effect of fine contents to hydraulic conductivity $k$, the calculated $k$ values were normalized by $c_f$ (Eq. (7) multiplied by $c$) and the results are presented in Fig. 6. From this plot, it is apparent that the increase in fine contents resulted to an exponential decrease in $k$ and the behavior manifested by each tailing samples is in good agreement with each other.

From the results presented, it can be stated that there is a similar and consistent relationship between $k$ and void ratio, $e$ as well as between $k$ and fine contents $F$ for the three (3) tailing samples. As such, it is convenient to express Eq. (9) as a single equation that will represent tailings of the same physical characteristics. Multiple regression analysis was performed to the combined experimental results of the tailing samples. The value of $k$ as a function of void ratio ($e$) and fine contents ($F$) is represented by Eq. (10) with coefficient of determination, $R^2 = 0.930$.

$$k = \left(4.78 \times 10^{-3}\right)\exp\left(-0.045F\left(e^{7.76}\right)\over 1+e\right)$$  (10)

The predictive capacity of Eq. (10) was compared with the measured $k$ from experimental data. The graph of log of predicted $k$ versus log of measured $k$ is shown in Fig. 7. The data corresponding to the measured $k$ against the predictive $k$ using Eq. (10) of the tailing samples appear closer to the equality line and less scattered. The $k$ values of TS#1 and TS#3 are located above the equality line indicating that the predicted model slightly under predicts the measured $k$ of TS#1 and TS#3. On the other hand, the $k$ values of TS#2 are below the equality line indicating that the model slightly over predicts the measured $k$ of TS#2. It can be stated that the proposed model (Eq. (10)) can be used to predict hydraulic conductivity, $k$ of fine tailings with non-plastic fines. However, Eq. (9) together with the values of model coefficients found in Table 2, are useful when dealing with a site specific investigation. The advantage of the developed equations is on its simplicity as it requires parameters (void ratio and fine contents) which can easily be determined; however, a note of caution must be made on its application. Hydraulic conductivity, $k$ values were determined from specimens that are homogenous and saturated. Application of equations to in situ conditions should be done with some care because intact tailing deposits are typically heterogeneous and stratified and may vary in every source and location.

Fig. 6 Predicted $k$ of mine tailings as a function of fine contents
equations are useful tools to predict hydraulic conductivity $k$ of materials which have similar geotechnical characteristics as tailings in this study and may be used to establish starting values when representative data are not readily available.

Several equations in the past relate the value of hydraulic conductivity $k$ in terms of its void ratio, $e$. It can be noted that some of these famous equations on hydraulic conductivity $k$ are in the form of the following general expression:

$$
k = f \left[ \frac{e^\alpha}{(1+e)^\beta} \right]
$$

For comparison purposes, the typical values of $\alpha$ and $\beta$ are presented in Table 3.

**Table 3** Typical values of parameters $\alpha$ and $\beta$ in Eq. (11)

<table>
<thead>
<tr>
<th>$\alpha$</th>
<th>$\beta$</th>
<th>Source</th>
<th>Application</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>0</td>
<td>Terzaghi [11]</td>
<td>for large-grained sand</td>
</tr>
<tr>
<td>3.8</td>
<td>1</td>
<td>Stone et al. [12]</td>
<td>Gold mine tailings</td>
</tr>
<tr>
<td>5.16</td>
<td>1</td>
<td>Kozeny-Carman modified by Aubertin, et al. [3]</td>
<td>Homogenized rock mine tailings</td>
</tr>
<tr>
<td>2.8</td>
<td>1</td>
<td>Shamsai, et al. [2]</td>
<td>Copper mine tailings with more than 50% fines</td>
</tr>
<tr>
<td>7.76</td>
<td>1</td>
<td>this study</td>
<td>Non-plastic, fine tailings</td>
</tr>
</tbody>
</table>

Fig. 7 Measured vs predicted $k$ values of mine tailings

It was also intended to carry out an investigation of the capability of proposed model by comparing the predicted $k$ values using Eq. (10) with the values estimated from other studies in the literature. Equations to determine $k$ developed by Hazen [7], Bates and Wayment [10], Kozeny-Carman modified by Aubertin [3] and Shamsai, et al. [2] were considered. The parameters used in the analysis, which are obtained from the average of the measured values of the 3 tailing samples, are the following: $D_{10} = 0.031$ mm, $D_{50} = 0.097$ mm, $C_u = 26.83$, and void ratio ranging from 0.60 to 1.1. Fig. 8 shows the comparison of predicted $k$ with values derived from literature. As can be seen from the graph, the modified Kozeny-Carman equation shows the closest correlation with the proposed equation specifically for tailings with seventy (70%) fine contents. Bates and Wayment equation exhibited the same trend as the predicted $k$ and seems to represent tailings with more than 70% fines. Hazen’s equation does not depend on void ratio; hence it yields a constant value of $9.6 \times 10^{-4}$ cm/sec for the above-stated parameters and corresponds to the average value of predicted $k$ with $F=0$. Shamsai’s equation gives the lowest $k$ values and appears to be far from the range of predicted $k$ values.
6. CONCLUSION

Series of laboratory tests were conducted to determine the geotechnical characteristics of tailings found in the Philippines namely, tailings from aggregate quarry in Cavite and tailings from gold mine sites in Davao and Masbate. Based on the obtained experimental results, the following conclusions were drawn:

The three tailing samples are considered as fine-grained consisting of fine sands and silts. Tailings from aggregate quarry (TS#1) is classified as poorly graded sand with silt having USCS symbol of SP-SM, gold tailings from Davao (TS#2) and Masbate (TS#3) are both classified as silty sand with USCS symbol of SM. The three tailing samples are non-plastic with a compressibility rating of almost none to slight. Microfabric analyses performed using electron microscopy showed a microstructure that is granular with silt-size grains.

Falling head permeability tests are performed on reconstituted samples compacted at varying void ratio and fine content. The hydraulic conductivity, $k$ of tailings falls within the range of $10^{-3}$ to $10^{-6}$ cm/sec. These tailings are considered to have low to very low permeability typical to silty sand or silty clay. Data obtained from test results showed that the values of $k$ exponentially increase as the void ratio increases, but exponentially decrease as fine contents increase. A new relation for predicting hydraulic conductivity of non-plastic fine tailings was formulated. The proposed empirical model is capable to taking into consideration the effect of void ratio and fines contents on the hydraulic conductivity. The advantage of the developed model is on its simplicity as it requires parameters (void ratio and fine contents) which can be easily determined. Comparison of the proposed model with existing models shows significantly better agreement of the proposed model with experimental data. The formulated model is a useful tool to estimate the hydraulic conductivity $k$ of non-plastic fine tailings, a parameter needed in seepage analysis, when these waste materials are used as embankment materials in the construction of tailing dams.

7. REFERENCES


